2.3 The Product Rule!

Ex.1 Find the derivative of $h(x)= (x^2+1)(x^3-2)$

$h(x) = x^5 - 2x^2 + x^3 - 2$

$h'(x) = 5x^4 - 4x + 3x^2$

The Product Rule:
If $p(x) = f(x)g(x)$, then $p'(x) = f'(x)g(x) + f(x)g'(x)$.

Leibniz:
If u and v are functions of x, $\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$

Ex.1 (redo) using the Product Rule.

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Proof of Product Rule

$F(x)=f(x)g(x)$

Need to "FOLLOW" NOT REPRODUCE

$$F'(x)= \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \left[\frac{f(x+h)-f(x)}{h} g(x+h) + f(x)\frac{g(x+h)-g(x)}{h}\right]$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \lim_{h\to 0} g(x+h) + \lim_{h\to 0} f(x) \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= f'(x)g(x) + f(x)g'(x)$$

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Ex. 2 Find $\frac{dy}{dx}$ if $y = (2x^3+5)(3x^2-x)$

$f(x) = g(x)p(x)$

$f'(x) = g'(x)p(x) + g(x)p'(x)$

$\frac{dy}{dx} = 6x^2(3x^2-x) + (2x^3+5)(6x-1)$

$= 18x^4 - 6x^3 + 12x^4 - 2x^3 + 30x - 5$

$= 30x^4 - 8x^3 + 30x - 5$

Ex.3 Differentiate $f(x) = \sqrt{x}(2-3x)$ and simplify.

$= x^{\frac{1}{2}}(2-3x)$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(2-3x) + x^{\frac{1}{2}}(-3)$

$= x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} - 3x^{\frac{1}{2}}$

$= x^{-\frac{1}{2}} - \frac{9}{2}x^{\frac{1}{2}}$

$= \frac{1}{x^{\frac{1}{2}}} - \frac{9}{2}x^{\frac{1}{2}}$

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Ex.4 Find the slope of the tangent to the graph of the function $f(x) = (3x^2+2)(2x^3-1)$ at the point (1,5).

① Find derivative

② Plug in $x=1$ for m_{tan}

$f'(x) = 6x(2x^3-1) + 6x^2(3x^3+2)$

$= 12x^4 - 6x + 18x^4 + 12x^2$

$= 30x^4 + 12x^2 - 6x$

Sub in $x=1$

$f'(1) = 30 + 12 - 6$

$= 36$

$\therefore m_{tan} = 36$

Extended Product Rule for a product of three functions

NOTE: If $p(x)=f(x)g(x)h(x)$ then

$p'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

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Hmk.
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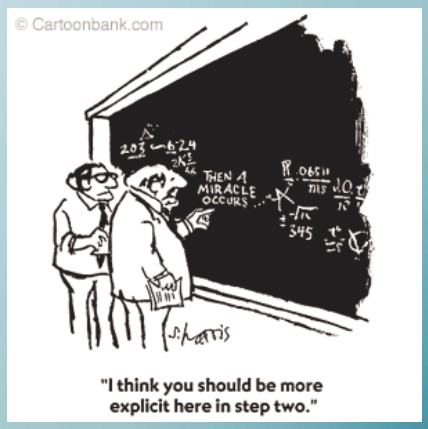

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