2.6 Applications of Quadratics

Ex 1 A football player kicks a ball of a football tee. The height of the ball, h, in metres after t seconds can be modelled using the formula: $h=-5t^2+20t$

a) What is the maximum height of the ball?

$h=-5(t^2-4t+4)+20$
$=-5(t-2)^2+20$

∴ Maximum is 20m.

b) What is the initial height of the ball?

$t=0$ $h=-5(0)^2+20(0)$
$=0$

∴ Initial height is zero

c) When does the ball hit the ground?

$h=0$ $h=-5t^2+20t$
$0=-5t(t-4)$
$t=0$ & $t=4$

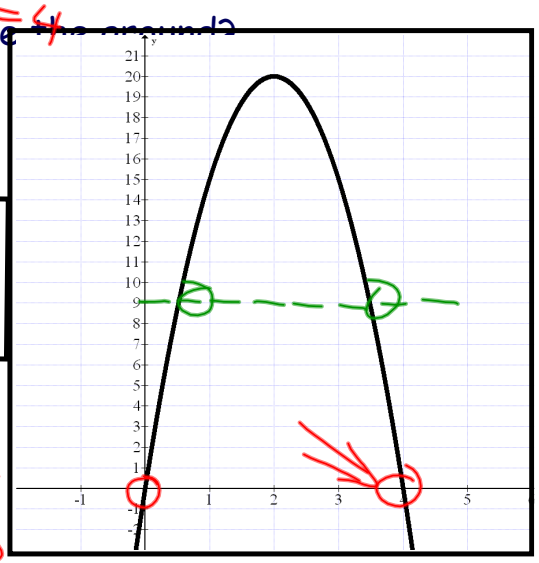
∴ hits the ground @ $t=4$

d) When is the ball more than 10 m above the ground?

$10=-5(t-2)^2+20$
$0=-5(t-2)^2+10$
$5(t-2)^2=10$
$(t-2)^2=2$
$t-2=\pm\sqrt{2}$
$t=2\pm\sqrt{2}$

∴ Ball is above 10m from $2-\sqrt{2}$ to $2+\sqrt{2}$

Graph



Ex 2 Playing Football on Mars

The force of gravity on Mars is less than half that on Earth. A ball thrown upward can be modelled using $h = -2t^2 + 15t + 2$ where h is the height in m and t is the time in seconds.

a) What is the initial height of the ball?

$t=0$ .... so initial height is 2

b) What is the maximum height of the ball?
When does the ball reach its maximum height?

$h = -2t^2 + 15t + 2$

$= -2\left(t^2 - \frac{15}{2}t + \frac{225}{16}\right) + \frac{225}{8} + 2$

$= -2\left(t - \frac{15}{4}\right)^2 + \frac{241}{8}$

∴ Max h is $\frac{241}{8}$ m @ $x = \frac{15}{4}$ s

c) When does the ball hit the ground?

$0 = -2t^2 + 15t + 2$

$t = \frac{-15 \pm \sqrt{15^2 - 4(-2)(2)}}{2(-2)}$

$= \frac{-15 \pm \sqrt{241}}{-4}$

$= \frac{15 + \sqrt{241}}{4}$ & $= \frac{15 - \sqrt{241}}{4}$

$= 7.63$s $= -0.13$s

Ball hits @ t=7.6s

d) When is the ball more than 20 m above the ground?

$h = 20$

$20 = -2t^2 + 15t + 2$

$0 = -2t^2 + 15t - 18$

$= (t-6)(2t-3)$

$t = 6$ & $t = \frac{3}{2}$

∴ Ball is over 20m from $t = \frac{3}{2}$s to 6s
1.5s to 6s

Graph

h

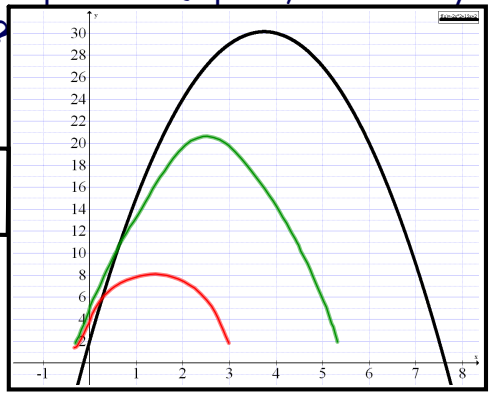
e) If the same ball was thrown upward on the Earth, describe how the relationship/equation will change?

- Zeros will be close
- Max would be lower

"a" be bigger

f) The force of gravity on Jupiter is much greater than on the Earth. If the same ball was thrown upward on Jupiter, how would you expect the relationship to change?

Graph



Ex. 3 Money, Money, Money

A study of the finances of Dominion Motors has shown that the profit of the company can be described by:

$$P = -2(n - 200)^2 + 450\,000$$

Where P represents the profit and n represents the number of cars sold.

a) What is the maximum profit?

The max. profit is $450 000.

b) How many cars have to be sold to reach the maximum profit?

∴ They have to sell 200 cars.

c) If they sell no cars what happens to their profit?

P = ?
n = 0

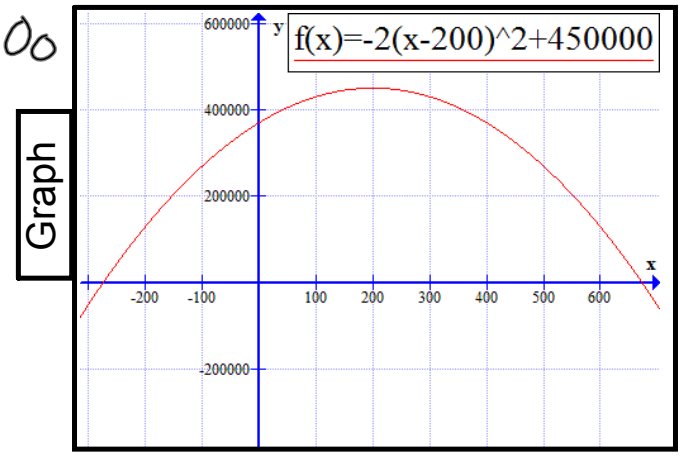
$P = -2(0-200)^2 + 450\,000$
$P = -2(40000) + 450000$
$P = 370000$
∴ Their profit is $370000

d) What is the minimum number of cars they need to sell to make a profit?

P =

e) Is there ever a point when they could sell too many cars?

$0 = -2(x-200)^2 + 450000$
$-450000 = -2(x-200)^2$
$225000 = (x-200)^2$
$\pm\sqrt{225000} = x - 200$
$200 \pm \sqrt{225000} = x$
$674, -274 = x$



∴ After 674 cars... they make no money.

HOMEWORK
-Handout 2.6